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Existence results for second-order dynamic inclusion with m-point boundary value conditions on time scales[☆]

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Abstract

In this work, the existence of solutions of m-point boundary problems for second-order dynamic inclusions on time scales is investigated. The main approach used here is based on a fixed point theorem due to Sadovskii together with a continuous selection theorem for lower semi-continuous multi-valued maps.

Keywords: Dynamic inclusions; Time scales; Boundary value; Fixed point; Lower semi-continuous multi-valued maps

1. Introduction

Let $\mathbb{T}$ be a time scale which has the subspace topology inherited from the standard topology on $\mathbb{R}$. For each interval I of $\mathbb{R}$, we define $I_{\mathbb{T}} = I \cap \mathbb{T}$. Recently, much attention has been paid to the existence of solutions of single-valued dynamic equations with boundary conditions on time scales, for example, Agarwal and O'Regan [1], Agarwal et al. [2], Bohner and Peterson [4], Erbe and Peterson [6,7], Henderson [8], Li and Sun [9], and Sun and Li [12]. Very recently, Merdivenci Atici and Biles in [10] established an existence result for first-order dynamic inclusions on time scales by using the method of lower and upper solutions combined with a fixed point theorem for condensing maps due to Martelli. However, to the best of our knowledge, few papers have been devoted to the existence of solutions of m-point boundary value problems for second-order dynamic inclusions on time scales. Motivated by the above mentioned work, in this work, we investigate the existence of solutions of the following second-order dynamic inclusions on time scales with m-point boundary conditions:

$$y^{\Delta\nabla}(t) \in F(y(t)), \quad t \in [0, b]_{\mathbb{T}}, \tag{1.1}$$

$$y^{\Delta}(0) = \sum_{i=1}^{m-2} a_i y^{\Delta}(\zeta_i), \qquad y(b) = \sum_{i=1}^{m-2} b_i y(\zeta_i), \tag{1.2}$$

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where $\sum_{i=1}^{m-2} a_i \neq 1, \sum_{i=1}^{m-2} b_i \neq 1$, $F : \mathbb{R} \to \mathcal{P}(\mathbb{R})$ is a given multi-valued map and $\mathcal{P}(\mathbb{R})$ is the family of all nonempty subsets of $\mathbb{R}$, $\zeta_i \in [0, \rho(b)]_{\mathbb{T}}$ with $0 < \zeta_1 < \zeta_2 < \cdots < \zeta_{m-2} < \rho(b)$, ρ is called the backward jump operator which will be defined later.

By Sadovskii's fixed point theorem together with a continuous selection theorem for lower semi-continuous multi-valued maps with closed convex values, we establish some existence results for the above problem (1.1) and (1.2).

2. Preliminaries

In this section, we recall some notation, definitions and lemmas that will be used in the sequel.

Definition 2.1. Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators $\sigma, \rho : \mathbb{T} \to \mathbb{T}$ are defined, respectively, by

$$\sigma(t) = \inf\{s > t : s \in \mathbb{T}\} \quad \text{and} \quad \rho(t) = \sup\{s < t : s \in \mathbb{T}\}.$$

In this definition we put $\inf \emptyset = \sup \mathbb{T}$ and $\sup \emptyset = \inf \mathbb{T}$. A point $t \in \mathbb{T}$ is called left-dense if $t > \inf \mathbb{T}$ and $\rho(t) = t$, left-scattered if $\rho(t) < t$, right-dense if $t < \sup \mathbb{T}$ and $\sigma(t) = t$, right-scattered if $\sigma(t) > t$. If $\mathbb{T}$ has a left-scattered maximum t_1, then $\mathbb{T}^\kappa = \mathbb{T} \setminus \{t_1\}$; otherwise $\mathbb{T}^\kappa = \mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum t_2, then $\mathbb{T}_\kappa = \mathbb{T} \setminus \{t_2\}$; otherwise $\mathbb{T}_\kappa = \mathbb{T}$.

A function $f : \mathbb{T} \to \mathbb{R}$ is left-dense continuous provided it is continuous at left-dense points in $\mathbb{T}$ and its right-sided limits exist at right-dense points in $\mathbb{T}$.

Definition 2.2. For $y : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}^\kappa$, we define the "delta derivative" of $y(t)$, $y^\Delta(t)$, to be the number (if it exists) with the property that for a given $\varepsilon > 0$, there exists a neighborhood $\mathcal{N}$ of t such that

$$|[y(\sigma(t)) - y(s)] - y^\Delta(t)[\sigma(t) - s]| < \varepsilon|\sigma(t) - s|$$

for all $s \in \mathcal{N}$. For $y : \mathbb{T} \to \mathbb{R}$ and $t \in \mathbb{T}_\kappa$, we define the "nabla derivative" of $y(t)$, $y^\nabla(t)$, to be the number (if it exists) with the property that for a given $\varepsilon > 0$, there exists a neighborhood $\mathcal{U}$ of t such that

$$|[y(\rho(t)) - y(s)] - y^\nabla(t)[\rho(t) - s]| < \varepsilon|\rho(t) - s|$$

for all $s \in \mathcal{U}$.

Throughout this work, we assume $\mathbb{T}$ is a nonempty closed subset (time scale) of $\mathbb{R}$ with $0 \in \mathbb{T}_\kappa$, $b \in \mathbb{T}^\kappa$.

Definition 2.3. If $H^\Delta(t) = h(t)$, then we define the delta integral by

$$\int_a^t h(s)\Delta s = H(t) - H(a).$$

If $\Phi^\nabla(t) = \phi(t)$, then we define nabla integral by

$$\int_a^t \phi(s)\nabla s = \Phi(t) - \Phi(a).$$

For more detailed results of time scales calculus, we refer the reader to [4].

Let $\mathbb{C} = C_{\mathrm{ld}}([0, b]_{\mathbb{T}}, \mathbb{R})$ be the Banach space composed of left-dense continuous functions from $[0, b]_{\mathbb{T}}$ into $\mathbb{R}$ with the norm

$$\|y\| := \sup\{|y(t)| : t \in [0, b]_{\mathbb{T}}\}.$$

Let $(X, |\cdot|)$ be a Banach space. Then a multi-valued map $\Theta : X \to \mathcal{P}(X)$ is convex (closed) valued if $\Theta(x)$ is convex (closed) for all $x \in X$. Θ is bounded on bounded sets if $\Theta(B) = \cup_{x \in B} \Theta(x)$ is bounded in X for any bounded set B of X (i.e. $\sup_{x \in B}\{\sup\{|y| : y \in \Theta(x)\}\} < \infty$).

$\Theta : \Omega \to \mathcal{P}(X)$ is said to be lower semi-continuous, l.s.c. for short, if $\Theta^{-1}(V)$ is open in Ω whenever $V \subset X$ is open.

Let $\Theta : \Omega \to \mathcal{P}(X)$ be a multi-valued map and $\theta : \Omega \to X$ be a single-valued function; if $\forall x \in \Omega, \theta(x) \in \Theta(x)$, then θ is called a selection function of Θ. If in addition, θ is continuous, then θ is called a continuous selection.

For more details on multi-valued maps, see the books of Deimling [5].

Definition 2.4. A function y is said to be a solution of (1.1) and (1.2) if $y : \mathbb{T} \to \mathbb{R}$ is delta differentiable, $y^{\Delta} : \mathbb{T}^{\kappa} \to \mathbb{R}$ is nabla differentiable on $\mathbb{T}^{\kappa} \cap \mathbb{T}_{\kappa}$, and y satisfies the dynamic inclusion (1.1) a.e. on $[0, b]_{\mathbb{T}}$ and the conditions (1.2).

The following lemmas are crucial in the proof our main result.

Lemma 2.1 (*[5, Lemma 2.1., pp. 14]*). *Let $\Omega \neq \emptyset$ be a subset of a Banach space, X be a Banach space, and $\Theta : \Omega \to \mathcal{P}(X)$ be l.s.c. with closed convex values. Then, given $(w_0, x_0) \in \operatorname{graph}(\Theta)$, Θ has a continuous selection θ such that $\theta(w_0) = x_0$.*

Lemma 2.2 (*[11, Sadovskii's Fixed Point Theorem]*). *Let P be a condensing operator on a Banach space X, i.e., P is continuous and takes bounded sets into bounded sets, and $\alpha(P(B)) < \alpha(B)$ for every bounded set B of X with $\alpha(B) > 0$. If $P(D) \subset D$ for a convex, closed and bounded set D of X, then P has a fixed point in D (where $\alpha(\cdot)$ denotes Kuratowski's measure of non-compactness).*

We remark here that a completely continuous operator is a condensing map.

Finally, we recall the definition of the Kuratowski measure of non-compactness [3]. Let B a bounded subset of a Banach space X. Then,

$$\alpha(B) = \inf\left\{\varepsilon > 0 : B \subset \bigcup_{i=1}^{m} M_i \text{ and } \operatorname{diam}(M_i) \leq \varepsilon\right\}.$$

3. Existence results

Let us list the following assumptions:

(A1) Let $F : \mathbb{R} \to \mathcal{P}(\mathbb{R})$ be a nonempty, closed and convex multi-valued map such that $u \mapsto F(u)$ is l.s.c. for $u \in \mathbb{R}$.

(A2) There exists a continuous nondecreasing function $\varphi : [0, \infty) \to (0, \infty)$ such that

$$\|F(u)\| := \sup\{|v| : v \in F(u)\} \leq \varphi(|u|)$$

for each $u \in \mathbb{R}$.

(A3) $\liminf_{q\to+\infty} \frac{\varphi(q)}{q} = \lambda < \infty$.

We first consider the following 'linear' boundary problem:

$$y^{\Delta\nabla}(t) = g(t), \quad t \in [0, b]_{\mathbb{T}}, \tag{3.1}$$

$$y^{\Delta}(0) = \sum_{i=1}^{m-2} a_i y^{\Delta}(\zeta_i), \qquad y(b) = \sum_{i=1}^{m-2} b_i y(\zeta_i). \tag{3.2}$$

For this 'linear' boundary problem (3.1) and (3.2), we have the following lemma.

Lemma 3.1. *If $\sum_{i=1}^{m-2} a_i \neq 1$ and $\sum_{i=1}^{m-2} b_i \neq 1$, $g \in C$, then $y(t)$ is a solution of the problem (3.1) and (3.2) if and only if*

$$y(t) = \int_0^t (t-s)g(s)\nabla s + t\,\frac{\sum\limits_{i=1}^{m-2} a_i \int_0^{\zeta_i} g(s)\nabla s}{1 - \sum\limits_{i=1}^{m-2} a_i} + \frac{1}{1 - \sum\limits_{i=1}^{m-2} b_i} \times \left[\sum_{i=1}^{m-2} b_i \int_0^{\zeta_i} (\zeta_i - s)g(s)\nabla s - \int_0^b (b-s)g(s)\nabla s + \left(\sum_{i=1}^{m-2} b_i\zeta_i - b\right)\frac{\sum\limits_{i=1}^{m-2} a_i \int_0^{\zeta_i} g(s)\nabla s}{1 - \sum\limits_{i=1}^{m-2} a_i}\right]. \tag{3.3}$$

Proof. In fact, if $y(t)$ is a solution of the problem (3.1) and (3.2), then by nabla integrating (3.1) from 0 to t we have

$$y^{\Delta}(t) - y^{\Delta}(0) = \int_0^t g(s)\nabla s. \tag{3.4}$$

Delta integrating (3.4) from 0 to t we obtain

$$y(t) = y(0) + t y^{\Delta}(0) + \int_0^t (t-s) g(s)\nabla s. \tag{3.5}$$

Taking $t = \zeta_i, i = 1, 2, \dots, m-2$ in (3.4), we get

$$y^{\Delta}(\zeta_i) = y^{\Delta}(0) + \int_0^{\zeta_i} g(s)\nabla s, \quad i = 1, 2, \dots, m-2. \tag{3.6}$$

Using the boundary condition $y^{\Delta}(0) = \sum_{i=1}^{m-2} a_i y^{\Delta}(\zeta_i)$ and (3.6), we have

$$y^{\Delta}(0) = \frac{\sum\limits_{i=1}^{m-2} a_i \int_0^{\zeta_i} g(s)\nabla s}{1 - \sum\limits_{i=1}^{m-2} a_i}. \tag{3.7}$$

Setting $t = b, \zeta_i, i = 1, 2, \dots, m-2$ in (3.5), we get

$$y(b) = y(0) + b y^{\Delta}(0) + \int_0^b (b-s) g(s)\nabla s \tag{3.8}$$

and

$$y(\zeta_i) = y(0) + \zeta_i y^{\Delta}(0) + \int_0^{\zeta_i} (\zeta_i - s) g(s)\nabla s, \quad i = 1, 2, \dots, m-2. \tag{3.9}$$

The boundary condition $y(b) = \sum_{i=1}^{m-2} b_i y(\zeta_i)$ and (3.8) and (3.9) yield that

$$\begin{aligned} y(0) = \frac{1}{1 - \sum\limits_{i=1}^{m-2} b_i} \Bigg[& \sum_{i=1}^{m-2} b_i \int_0^{\zeta_i} (\zeta_i - s) g(s)\nabla s - \int_0^b (b-s) g(s)\nabla s \\ & + \left(\sum_{i=1}^{m-2} b_i \zeta_i - b \right) \frac{\sum\limits_{i=1}^{m-2} a_i \int_0^{\zeta_i} g(s)\nabla s}{1 - \sum\limits_{i=1}^{m-2} a_i} \Bigg]. \end{aligned} \tag{3.10}$$

Substituting (3.7)–(3.10) into (3.5), we see that $y(t)$ satisfies (3.3) on $[0, b]_{\mathbb{T}}$.

If $y(t)$ is defined by (3.3) on $[0, b]_{\mathbb{T}}$, using Theorem 8.50 (iii) in [4] it is easy to verify that $y(t)$ satisfies (3.1) and (3.2) on $[0, b]_{\mathbb{T}}$.

We are now in a position to state and prove our existence results for the problem (1.1) and (1.2). □

Theorem 3.1. *Assume* (A1)–(A3) *hold. Then the problem* (1.1) *and* (1.2) *has at least one solution on* $[0, b]_{\mathbb{T}}$*, provided that*

$$(b^2 + b\alpha + \beta + \gamma + \eta\alpha)\lambda < 1,$$

where

$$\alpha = \frac{\sum_{i=1}^{m-2} |a_i|\zeta_i}{\left|1 - \sum_{i=1}^{m-2} a_i\right|}, \qquad \beta = \frac{\sum_{i=1}^{m-2} |b_i|\zeta_i^2}{\left|1 - \sum_{i=1}^{m-2} b_i\right|}, \qquad \gamma = \frac{b^2}{\left|1 - \sum_{i=1}^{m-2} b_i\right|}, \qquad \eta = \frac{\left|\sum_{i=1}^{m-2} b_i\zeta_i - b\right|}{\left|1 - \sum_{i=1}^{m-2} b_i\right|}.$$

Proof. Note that (A1) and Lemma 2.1 imply F is lower semi-continuous. Then, from Lemma 2.1, there exists a continuous function $f : \mathbb{C} \to \mathbb{R}$ such that $f(y) \in F(y)$ for all $y \in \mathbb{C}$. We consider the problem

$$y^{\Delta\nabla}(t) = f(y(t)), \quad t \in [0, b]_{\mathbb{T}}, \tag{3.11}$$

$$y^{\Delta}(0) = \sum_{i=1}^{m-2} a_i y^{\Delta}(\zeta_i), \qquad y(b) = \sum_{i=1}^{m-2} b_i y(\zeta_i). \tag{3.12}$$

It is clear that if y is a solution of the problem (3.11) and (3.12), then y is a solution to the problem (1.1) and (1.2). Consider the operator $N : \mathbb{C} \to \mathbb{C}$, defined by

$$\begin{aligned} N(y)(t) = {} & \int_0^t (t-s) f(y(s))\nabla s + t\frac{\sum_{i=1}^{m-2} a_i \int_0^{\zeta_i} f(y(s))\nabla s}{1 - \sum_{i=1}^{m-2} a_i} \\ & + \frac{1}{1 - \sum_{i=1}^{m-2} b_i}\left[\sum_{i=1}^{m-2} b_i \int_0^{\zeta_i} (\zeta_i - s) f(y(s))\nabla s - \int_0^b (b-s) f(y(s))\nabla s \right. \\ & \left. + \left(\sum_{i=1}^{m-2} b_i\zeta_i - b\right) \frac{\sum_{i=1}^{m-2} a_i \int_0^{\zeta_i} f(y(s))\nabla s}{1 - \sum_{i=1}^{m-2} a_i}\right], \quad t \in [0, b]_{\mathbb{T}}. \end{aligned}$$

Obviously, the fixed points of N are solutions to the problem (3.11) and (3.12). Next we shall prove that N is completely continuous. We break the proof into several steps.

Step 1. N is continuous.

Since the function f is continuous, this conclusion can be easily obtained.

Step 2. For each constant $q > 0$, let $B_q = \{y \in \mathbb{C} : \|y\| \le q\}$. Then B_q is a bounded closed convex set in $\mathbb{C}$. We claim that there exists a positive number q such that $N(B_q) \subseteq B_q$. If this is not true, then for each positive number q, there exists a function $y_q(\cdot) \in B_q$, but $\|N(y_q(t))\| > q$ for some $t \in [0, b]_{\mathbb{T}}$. However, on the other hand, we have

$$\begin{aligned} q & < \|N(y_q(t))\| \\ & \le b\int_0^b \varphi(\|y_q\|)\nabla s + b\frac{\sum_{i=1}^{m-2} |a_i| \int_0^{\zeta_i} \varphi(\|y_q\|)\nabla s}{\left|1 - \sum_{i=1}^{m-2} a_i\right|} \\ & \quad + \frac{1}{\left|1 - \sum_{i=1}^{m-2} b_i\right|}\left[\sum_{i=1}^{m-2} |b_i| \int_0^{\zeta_i} \zeta_i\varphi(\|y_q\|)\nabla s + b\int_0^b \varphi(\|y_q\|)\nabla s \right. \end{aligned}$$

$$+\left.\left|\sum_{i=1}^{m-2} b_i\zeta_i - b\right| \frac{\sum_{i=1}^{m-2}|a_i|\int_0^{\zeta_i}\varphi(\|y_q\|)\nabla s}{\left|1-\sum_{i=1}^{m-2}a_i\right|}\right]$$

$$\leq b\int_0^b \varphi(q)\nabla s + b\frac{\sum_{i=1}^{m-2}|a_i|\int_0^{\zeta_i}\varphi(q)\nabla s}{\left|1-\sum_{i=1}^{m-2}a_i\right|}$$

$$+\frac{1}{\left|1-\sum_{i=1}^{m-2}b_i\right|}\left[\sum_{i=1}^{m-2}|b_i|\int_0^{\zeta_i}\zeta_i\varphi(q)\nabla s + b\int_0^b\varphi(q)\nabla s + \left|\sum_{i=1}^{m-2}b_i\zeta_i - b\right|\frac{\sum_{i=1}^{m-2}|a_i|\int_0^{\zeta_i}\varphi(q)\nabla s}{\left|1-\sum_{i=1}^{m-2}a_i\right|}\right]$$

$$= (b^2+b\alpha+\beta+\gamma+\eta\alpha)\varphi(q).$$

Dividing both sides by q and take the lower limit as $q \to \infty$, we get

$$(b^2+b\alpha+\beta+\gamma+\eta\alpha)\lambda \geq 1,$$

which contradicts the relation $(b^2+b\alpha+\beta+\gamma+\eta\alpha)\lambda < 1$. Hence for some positive number q, we have $N(B_q) \subseteq B_q$.

Step 3. The family $\{Ny : y \in B_q\}$ is a family of equicontinuous functions.

Let $t_1, t_2 \in [0, b]_{\mathbb{T}}$, $t_1 < t_2$ and $B_q = \{y \in \mathbb{C} : \|y\| \leq q\}$ be a bounded set of $\mathbb{C}$. Then for each $t \in [0, b]_{\mathbb{T}}$ we have

$$N(y)(t) = \int_0^t (t-s)f(y(s))\nabla s + t\frac{\sum_{i=1}^{m-2}a_i\int_0^{\zeta_i}f(y(s))\nabla s}{1-\sum_{i=1}^{m-2}a_i}$$

$$+\frac{1}{1-\sum_{i=1}^{m-2}b_i}\left[\sum_{i=1}^{m-2}b_i\int_0^{\zeta_i}(\zeta_i-s)f(y(s))\nabla s - \int_0^b(b-s)f(y(s))\nabla s\right.$$

$$\left.+\left(\sum_{i=1}^{m-2}b_i\zeta_i - b\right)\frac{\sum_{i=1}^{m-2}a_i\int_0^{\zeta_i}f(y(s))\nabla s}{1-\sum_{i=1}^{m-2}a_i}\right].$$

Thus,

$$|N(y)(t_2)-N(y)(t_1)| \leq \left|\int_0^{t_1}(t_2-t_1)f(y(s))\nabla s\right| + \left|\int_{t_1}^{t_2}(t_2-s)f(y(s))\nabla s\right| + (t_2-t_1)\frac{\sum_{i=1}^{m-2}|a_i|\int_0^{\zeta_i}\varphi(q)\nabla s}{\left|1-\sum_{i=1}^{m-2}a_i\right|}$$

$$\leq (t_2-t_1)\int_0^{t_1}\varphi(q)\nabla s + t_2\int_{t_1}^{t_2}\varphi(q)\nabla s + (t_2-t_1)\frac{\sum_{i=1}^{m-2}|a_i|\int_0^{\zeta_i}\varphi(q)\nabla s}{\left|1-\sum_{i=1}^{m-2}a_i\right|}.$$

As $t_2 \to t_1$, the right-hand side of the above inequality is independent of $y \in B_q$ and tends to zero. Thus, the set $\{Ny : y \in B_q\}$ is equicontinuous.

As a consequence of Step 1 to Step 3 together with the Ascoli–Arzela theorem, we can conclude that $N : \mathbb{C} \to \mathbb{C}$ is completely continuous, and thus is a condensing map. By Lemma 2.2, we deduce that N has a fixed point which is a solution of the problem (1.1) and (1.2). □

From Theorem 3.1, we can obtain the following results.

Corollary 3.1. *Assume that* (A1) *holds. Suppose further that there exist real numbers* $c, d > 0$, $0 < \tau < 1$ *such that*

$$\|F(u)\| \leq c|u|^{\tau} + d, \quad u \in \mathbb{R}.$$

Then the problem (1.1) *and* (1.2) *has at least one solution on* $[0, b]_{\mathbb{T}}$.

Corollary 3.2. *Assume that* (A1)–(A2) *hold and the condition* (A3) *is replaced by*

(A3′) *There exists a positive real number* r *such that*

$$\frac{r}{\varphi(r)} \geq b^2 + b\alpha + \beta + \gamma + \eta\alpha,$$

where $\alpha, \beta, \gamma, \eta$ *are defined as in Theorem* 3.1*. Then the problem* (1.1) *and* (1.2) *has at least one solution on* $[0, b]_{\mathbb{T}}$.

Proof. Define $B_r = \{y \in \mathbb{C} : \|y\| \leq r\}$, where r satisfies the condition (A3′). We can see that, for each $y \in B_r$, one has $N(y) \in B_r$, where the operator N is defined as in Theorem 3.1. Hence we have $N(B_r) \subseteq B_r$. We can also show that the family $\{Ny : y \in B_r\}$ is a family of equicontinuous functions. Now, by Lemma 2.2, we can conclude that the problem (1.1) and (1.2) has at least one solution on $[0, b]_{\mathbb{T}}$. □

Example 3.1. Let $\mathbb{T} = \mathbb{R}$, $a_i = b_i = 0$, $i = 1, \ldots, m-2$ and take $F(y(t)) = \{f(y(t))\}$, $t \in [0, 1]$. Thus the boundary value problem (1.1) and (1.2) reduces to

$$y''(t) = f(y(t)), \quad t \in [0, 1],$$
$$y'(0) = y(1) = 0.$$

If $f : \mathbb{R} \to \mathbb{R}$ is continuous and for each $u \in \mathbb{R}$, $|f(u)| \leq 1 + |u|^{\tau}$, $0 < \tau < 1$, then from Corollary 3.1, the above problem has a solution.

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